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AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Aircraft Structures Technical Memorandum 536

DETERMINATION OF SYMMETRIC END LOADS FROM BULK STRESSES ON A RECTANGULAR PRISM

by

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SUMMARY

A mathematical method is presented for the determination of the normal symmetric end loads from known values of the bulk stress on a lateral surface of a three dimensional rectangular prism. A FORTRAN computer program has been developed to implement this method, and results for an arbitrarily chosen case are presented.

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Victoria Australia

CONTENTS

SYMBOLS

x, y, z	Cartesian co-ordinate axes
$2a, 2b, 2c$	Length, depth, breadth of the prism
ν	Poisson's ratio
$\sigma_x, \sigma_y, \sigma_z$	Normal stress components in the x, y, z directions
$2k_1b, 2k_2c$	Lengths of loaded portion in the y and z directions
l, m, n	Indicies of the Fourier series
A_{mn}, B_{nl}, D_{lm}	Fourier coefficients
$f(y, z)$	Stress distribution function on the faces $x = \pm a$

1. INTRODUCTION

The stress distribution in rectangular prisms subjected to certain normal end forces on its two opposite faces is a fundamental problem in Applied Mechanics. The stress distribution near the loaded surface may be very complex, and the distribution given by elementary theory is often inaccurate [1]. In this paper, it is assumed that the bulk stress $(\sigma_x + \sigma_y + \sigma_z)$ on a lateral surface of a rectangular prism, which is being subjected to an arbitrary loading, is known (i.e. measurable). Such a measurement could be obtained using advanced thermal emission techniques. The aim of this paper is to reveal how the measured values of bulk stresses can be used to determine the actual end loads.

A method for calculating the bulk stresses produced by a self-equilibrating end load is given in [1]. This will be referred to as the Direct solution. In the Inverse method we will determine the end loading from a known distribution of bulk stress values on a lateral surface. In this case, bulk stresses on the lateral surface were simulated using the Direct solution. In this preliminary "proof of concept" work, the effect of experimental noise on the thermal signal was not considered.

This technique of determining the actual end load based on the known bulk stress field has many potential applications, i.e. in fatigue tests involving rectangular cross sections to determine the stress distribution inside the specimens. Other possible applications include the determination of the longitudinal stress distribution around a planar crack at right angles to the sides of the · rism.

2. DIRECT SOLUTION

The solution, given in [1], is first used to generate the bulk stresses on a lateral surface for the case of a rectangular prism subjected to end forces symmetric in x, y and z. For the rectangular prism shown in Figure 1, in the absence of the body forces, the general solution is as follows.

First define the Galerkin vector $\mathbf{F}$.

$$\mathbf{F} = \mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z \tag{1}$$

where

$$F_x = \sum_m \sum_n \frac{A_{mn}a}{\alpha_{mn}^3 \cosh \alpha_{mn}a}[\alpha_{mn}x \cosh \alpha_{mn}x - (2\nu + \alpha_{mn}a \coth \alpha_{mn}a) \sinh \alpha_{mn}x] \times$$
$$\times \cos \frac{m\pi y}{b} \cos \frac{n\pi z}{c}$$

$$F_y = \sum_n \sum_l \frac{B_{nl}b}{\beta_{nl}^3 \cosh \beta_{nl}b}[\beta_{nl}y \cosh \beta_{nl}y - (2\nu + \beta_{nl}b \coth \beta_{nl}b) \sinh \beta_{nl}y] \times$$
$$\times \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a}$$

$$F_z = \sum_l \sum_m \frac{D_{lm}c}{\gamma_{lm}^3 \cosh \gamma_{lm}c}[\gamma_{lm}z \cosh \gamma_{lm}z - (2\nu + \gamma_{lm}c \coth \gamma_{lm}c) \sinh \gamma_{lm}z] \times$$
$$\times \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b}$$

and

$$\alpha_{mn}^2 = \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{c}\right)^2$$
$$\beta_{nl}^2 = \left(\frac{n\pi}{c}\right)^2 + \left(\frac{l\pi}{a}\right)^2$$
$$\gamma_{lm}^2 = \left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

$$l, m, n = 0, 1, 2, 3...$$

The stress components are then related to $\mathbf{F}$ by the following equations.

$$\sigma_x = 2(1-\nu)\frac{\partial}{\partial x}\nabla^2 F_x + \left(\nu\nabla^2 - \frac{\partial^2}{\partial x^2}\right) \operatorname{div} \mathbf{F}, \tag{2}$$

$$\tau_{yz} = (1-\nu)\left(\frac{\partial}{\partial z}\nabla^2 F_y + \frac{\partial}{\partial y}\nabla^2 F_z\right) - \frac{\partial^2}{\partial y \partial z} \operatorname{div} \mathbf{F}. \tag{3}$$

This gives:

$$
\begin{aligned}
\sigma_x = & \sum_m \sum_n \frac{A_{mn} a}{\cosh \alpha_{mn} a} [(1 + \alpha_{mn} a \coth \alpha_{mn} a) \cosh \alpha_{mn} x - \alpha_{mn} x \sinh \alpha_{mn} x] \\
& \times \cos \frac{m\pi y}{b} \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{B_{nl} b}{\beta_{nl}^2 c^2 \cosh \beta_{nl} b} \left[2\nu n^2 \pi^2 \cosh \beta_{nl} y + \left(\frac{l\pi c}{a} \right)^2 \right. \\
& \left. \times [(1 - \beta_{nl} b \coth \beta_{nl} b) \cosh \beta_{nl} y + \beta_{nl} y \sinh \beta_{nl} y] \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \\
& + \sum_l \sum_m \frac{D_{lm} c}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm} c} \left[2\nu m^2 \pi^2 \cosh \gamma_{lm} z + \left(\frac{l\pi b}{a} \right)^2 [(1 - \gamma_{lm} c \coth \gamma_{lm} c) \right. \\
& \left. \times \cosh \gamma_{lm} z + \gamma_{lm} z \sinh \gamma_{lm} z] \right] \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b}
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\tau_{yz} = & - \sum_m \sum_n \frac{A_{mn} a m n \pi^2}{\alpha_{mn}^2 b c \cosh \alpha_{mn} a} \left[[1 - (2\nu + \alpha_{mn} a \coth \alpha_{mn} a)] \cosh \alpha_{mn} x \right. \\
& \left. + \alpha_{mn} x \sinh \alpha_{mn} x \right] \sin \frac{n\pi z}{c} \sin \frac{m\pi y}{b} + \sum_n \sum_l \frac{B_{nl} n \pi b}{\beta_{nl} c \cosh \beta_{nl} b} \left[\beta_{nl} y \cosh \beta_{nl} y \right. \\
& \left. - \beta_{nl} b \coth \beta_{nl} b \sinh \beta_{nl} y \right] \sin \frac{n\pi z}{c} \cos \frac{l\pi x}{a} + \sum_l \sum_m \frac{D_{lm} m \pi c}{\gamma_{lm} b \cosh \gamma_{lm} c} \\
& \times \left[\gamma_{lm} z \cosh \gamma_{lm} z - \gamma_{lm} c \coth \gamma_{lm} c \sinh \gamma_{lm} z \right] \sin \frac{m\pi y}{b} \cos \frac{l\pi x}{a}
\end{aligned}
\tag{5}
$$

Expressions for the other stress components can be written in a similar manner.

With reference to Figure 1, the following boundary conditions apply.

$$
\begin{array}{llll}
\text{on} \quad y = \pm b\,; & \sigma_y = 0, & \tau_{xy} = 0 \quad \text{and} & \tau_{yz} = 0, \\
\text{on} \quad z = \pm c\,; & \sigma_z = 0, & \tau_{xz} = 0 \quad \text{and} & \tau_{yz} = 0, \\
\text{on} \quad x = \pm a\,; & \sigma_x = f_1(y, z), & \tau_{xy} = 0 \quad \text{and} & \tau_{xz} = 0.
\end{array}
\tag{6}
$$

All the shear stress boundary conditions are exactly satisfied by (5) and similar expressions for τ_{xy} and τ_{xz}. The normal stress boundary conditions are approximately satisfied by equating low order Fourier cosine terms in equation (4), and in similar expressions for σ_y and σ_z. Taking the double Fourier cosine transforms of these three equations (with their approximated normal stress boundary conditions) gives the expressions (7),(8) and (9).

$$
\begin{aligned}
&\sum_m \frac{4A_{mn}\delta_1 \tanh \alpha_{mn}a(-1)^{l+m}}{\alpha_{mn}b(\alpha_{mn}^2 a^2 + l^2\pi^2)^2}\left[\nu\left(\frac{n\pi a}{c}\right)^2(\alpha_{mn}^2 a^2 + l^2\pi^2)\right.\\
&\left. + \left(\frac{m\pi a}{b}\right)^2 l^2\pi^2\right] + B_{nl}\delta_2\left(1 + \frac{2\beta_{nl}b}{\sinh 2\beta_{nl}b}\right)\\
&+ \sum_m \frac{4D_{lm}\delta_3 \tanh \gamma_{lm}c(-1)^{m+n}}{\gamma_{lm}b(\gamma_{lm}^2 c^2 + n^2\pi^2)^2}\left[\nu\left(\frac{l\pi c}{a}\right)^2(\gamma_{lm}^2 c^2 + n^2\pi^2)\right.\\
&\left. + \left(\frac{m\pi c}{b}\right)^2 n^2\pi^2\right] = 0
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
&\delta_1 = 2 \quad \text{when} \quad n = 0, \quad \delta_1 = 1 \quad \text{when} \quad n \neq 0\\
&\delta_2 = 2 \quad \text{when} \quad l = 0 \quad \text{or} \quad n = 0, \quad \delta_2 = 1 \quad \text{when} \quad l \neq 0, \quad n \neq 0\\
&\delta_3 = 2 \quad \text{when} \quad l = 0, \quad \delta_3 = 1 \quad \text{when} \quad l \neq 0
\end{aligned}
$$

$$
\begin{aligned}
&\sum_n \frac{4A_{mn}\delta_1 \tanh \alpha_{mn}a(-1)^{l+n}}{\alpha_{mn}c(\alpha_{mn}^2 a^2 + l^2\pi^2)^2}\left[\nu\left(\frac{m\pi a}{b}\right)^2(\alpha_{mn}^2 a^2 + l^2\pi^2)\right.\\
&\left. + \left(\frac{n\pi a}{c}\right)^2 l^2\pi^2\right] + \sum_n \frac{4B_{nl}\delta_2 \tanh \beta_{nl}b(-1)^{m+n}}{\beta_{nl}c(\beta_{nl}^2 b^2 + m^2\pi^2)^2}\\
&\times \left[\nu\left(\frac{l\pi b}{a}\right)^2(\beta_{nl}^2 b^2 + m^2\pi^2) + \left(\frac{n\pi b}{c}\right)^2 m^2\pi^2\right]\\
&+ D_{lm}\delta_3\left(1 + \frac{2\gamma_{lm}c}{\sinh 2\gamma_{lm}c}\right) = 0
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
&\delta_1 = 2 \quad \text{when} \quad m = 0, \quad \delta_1 = 1 \quad \text{when} \quad m \neq 0\\
&\delta_2 = 2 \quad \text{when} \quad l = 0, \quad \delta_2 = 1 \quad \text{when} \quad l \neq 0\\
&\delta_3 = 2 \quad \text{when} \quad m = 0 \quad \text{or} \quad l = 0, \quad \delta_3 = 1 \quad \text{when} \quad m \neq 0, \quad l \neq 0
\end{aligned}
$$

$$A_{mn}\delta_1\left(1+\frac{2\alpha_{mn}a}{\sinh 2\alpha_{mn}a}\right)+\sum_l \frac{4B_{nl}\delta_2 \tanh \beta_{nl}b(-1)^{l+m}}{\beta_{nl}a(\beta_{nl}^2 b^2+m^2\pi^2)^2}$$
$$\times\left[\nu\left(\frac{n\pi b}{c}\right)^2(\beta_{nl}^2 b^2+m^2\pi^2)+\left(\frac{l\pi b}{a}\right)^2 m^2\pi^2\right]$$
$$+\sum_l \frac{4D_{lm}\delta_3 \tanh \gamma_{lm}c(-1)^{l+n}}{\gamma_{lm}a(\gamma_{lm}^2 c^2+n^2\pi^2)^2}\left[\nu\left(\frac{m\pi c}{b}\right)^2(\gamma_{lm}^2 c^2+n^2\pi^2)\right.$$
$$\left.+\left(\frac{l\pi c}{a}\right)^2 n^2\pi^2\right]=I_{mn} \tag{9}$$

where

$$\delta_1=2 \quad \text{when} \quad m=0 \quad \text{or} \quad n=0, \quad \delta_1=1 \quad \text{when} \quad m\neq 0, \quad n\neq 0$$
$$\delta_2=2 \quad \text{when} \quad n=0, \quad \delta_2=1 \quad \text{when} \quad n\neq 0$$
$$\delta_3=2 \quad \text{when} \quad m=0, \quad \delta_3=1 \quad \text{when} \quad m\neq 0$$

and

$$I_{mn}=\frac{1}{abc}\int_{-b}^{+b}\int_{-c}^{+c} f_1(y,z)\cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{n\pi z}{c}\right)dy\,dz$$

Equations (7),(8) and (9) may then be solved simultaneously for the unknown Fourier coefficients. The stress at any point in the prism may then be calculated by substituting the Fourier coefficients into equations (4),(5) and similar.

Since the given equations are valid only for self equilibrating end loads, it is necessary to modify the required end loading $f(y,z)$ so that the net force acting on each end face is zero. This produces a modified loading distribution $f_1(y,z)$. For symmetric loading,

$$f_1(y,z)=\left[f(y,z)-\frac{P}{4bc}\right] \tag{10}$$

where P is the total load due to $f(y,z)$, i.e.

$$P=\int_{-b}^{+b}\int_{-c}^{+c} f(y,z)\,dy\,dz \tag{11}$$

The previous equations calculate the stresses due to the modified loading $f_1(y,z)$. The stress component σ_x, calculated from equation (4), is a Fourier series approximation to $f_1(y,z)$.

3. INVERSE SOLUTION

The main aim of this section is to provide an Inverse solution to the problem, which involves calculation of the end stress σ_x from the known bulk stress distribution ($\sigma_x + \sigma_y + \sigma_z$) on one lateral surface of the prism. Consider the case when we know the bulk stress distribution on the surface $y = +b$. On this surface, $\sigma_y = 0$, and so the bulk stress is $\sigma_x + \sigma_z$. From equation (4) we obtain the following expression for the bulk stress.

$$
\begin{aligned}
(\sigma_x + \sigma_z)\big|_{y=b} = {} & \sum_m \sum_n \frac{A_{mn} a(-1)^m}{\cosh \alpha_{mn} a} \Big[(1 + \alpha_{mn} a \coth \alpha_{mn} a) \cosh \alpha_{mn} x \\
& - \alpha_{mn} x \sinh \alpha_{mn} x \Big] \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{B_{nl} b}{\beta_{nl}^2 c^2 \cosh \beta_{nl} b} \\
& \times \left[2\nu n^2 \pi^2 \cosh \beta_{nl} b + \left(\frac{l\pi c}{a} \right)^2 [(1 - \beta_{nl} b \coth \beta_{nl} b) \cosh \beta_{nl} b \right. \\
& \left. + \beta_{nl} b \sinh \beta_{nl} b] \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} + \sum_l \sum_m \frac{D_{lm} c(-1)^m}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm} c} \\
& \times \left[2\nu m^2 \pi^2 \cosh \gamma_{lm} z + \left(\frac{l\pi b}{a} \right)^2 [(1 - \gamma_{lm} c \coth \gamma_{lm} c) \cosh \gamma_{lm} z \right. \\
& \left. + \gamma_{lm} z \sinh \gamma_{lm} z] \right] \cos \frac{l\pi x}{a} + \sum_l \sum_m \frac{D_{lm} c(-1)^m}{\cosh \gamma_{lm} c} \Big[(1 + \gamma_{lm} c \coth \gamma_{lm} c) \\
& \times \cosh \gamma_{lm} z - \gamma_{lm} z \sinh \gamma_{lm} z \Big] \cos \frac{l\pi x}{a} + \sum_m \sum_n \frac{A_{mn} a(-1)^m}{\alpha_{mn}^2 b^2 \cosh \alpha_{mn} a} \\
& \times \left[2\nu m^2 \pi^2 \cosh \alpha_{mn} x + \left(\frac{n\pi b}{c} \right)^2 [(1 - \alpha_{mn} a \coth \alpha_{mn} a) \cosh \alpha_{mn} x \right. \\
& \left. + \alpha_{mn} x \sinh \alpha_{mn} x] \right] \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{B_{nl} b}{\beta_{nl}^2 a^2 \cosh \beta_{nl} b} \\
& \times \left[2\nu l^2 \pi^2 \cosh \beta_{nl} b + \left(\frac{n\pi a}{c} \right)^2 [(1 - \beta_{nl} b \coth \beta_{nl} b) \cosh \beta_{nl} b \right. \\
& \left. + \beta_{nl} b \sinh \beta_{nl} b] \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \qquad (12)
\end{aligned}
$$

By taking the Fourier transform of this equation, as shown in Appendix A, the following results are obtained.

$$
\begin{aligned}
&\frac{1}{abc}\int_{-c}^{+c}\int_{-a}^{+a}(\sigma_x+\sigma_z)\big|_{y=b}\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{n\pi z}{c}\right)\,dx\,dz \\
&\qquad=\sum_m \frac{4A_{mn}(-1)^{m+l}(1+\delta_{on})\tanh\alpha_{mn}a}{b\left(\alpha_{mn}^2+(l\pi/a)^2\right)^2} \\
&\qquad\qquad\times\left[\alpha_{mn}^3+\frac{1}{\alpha_{mn}b^2}\left\{\nu m^2\pi^2\left(\alpha_{mn}^2+\left(\frac{l\pi}{a}\right)^2\right)+\left(\frac{n\pi b}{c}\right)^2\left(\frac{l\pi}{a}\right)^2\right\}\right] \\
&\qquad+\sum_m \frac{4D_{lm}(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{b\left(\gamma_{lm}^2+(n\pi/c)^2\right)^2} \\
&\qquad\qquad\times\left[\gamma_{lm}^3+\frac{1}{\gamma_{lm}b^2}\left\{\nu m^2\pi^2\left(\gamma_{lm}^2+\left(\frac{n\pi}{c}\right)^2\right)+\left(\frac{l\pi b}{a}\right)^2\left(\frac{n\pi}{c}\right)^2\right\}\right] \\
&\qquad+\frac{B_{nl}\pi^2}{\beta_{nl}^2a^2c^2}(1+\delta_{on})(1+\delta_{ol})(a^2n^2+c^2l^2) \\
&\qquad\qquad\times(2\nu+1-\beta_{nl}b\coth\beta_{nl}b+\beta_{nl}b\tanh\beta_{nl}b)
\end{aligned}
\tag{13}
$$

where

$$
\delta_{on}=\begin{cases}1 & \text{when } n=0\\ 0 & \text{when } n\neq 0\end{cases}
$$

$$
\delta_{ol}=\begin{cases}1 & \text{when } l=0\\ 0 & \text{when } l\neq 0\end{cases}
$$

Equation (13) may then be applied for each value of n and l to give a set of linear simultaneous equations with the Fourier coefficients A_{mn}, B_{nl} and D_{lm} as the unknowns. Since the boundary conditions of equation (6) still apply, equations (7) and (8) are also used in conjunction with (13). In effect, equation (13) replaces equation (9) for the Inverse solution. This imposes the restriction that $l=m$, so that the number of linear equations produced by (7) (8) and (13) equals the number of unknown Fourier coefficients. The resulting set of equations is then solved to produce a set of Fourier coefficients A_{mn}, B_{nl} and D_{lm} for the Inverse problem. Equation (4) is then used to obtain the applied stress σ_x.

4. RESULTS AND DISCUSSION

4.1 General

The Direct and Inverse programs were implemented using FORTRAN 77 on an Apollo DN10000 computer. A variety of numerical procedures were considered for performing the integrations required by equations (9) and (13). Initial versions of the programs used either trapezoidal or Simpson's rule. It was found that these methods were not sufficiently accurate, and the Gauss-Quadrature method was finally selected. Reference [2] describes the Gauss-Quadrature technique in detail.

Various methods were also used for the solution of the resulting set of linear simultaneous equations. Gaussian elimination with full pivoting was employed in the final program.

Both the Direct and Inverse programs were written with the aim of minimising numerical errors. Double precision was used throughout the programs, and many mathematical functions were re-written in more suitable forms to reduce errors for large values of l, m and n.

4.2 Direct Results

The equations given in the previous section are applicable to a prism with an arbitrary rectangular cross section. However, for illustrative purposes, a square cross section with a square loading area was considered. In this case, $b = c = 2$ and $k_1 = k_2 = 0.5$. With reference to Figure 1, a stress of $f(y,z) = 16$ was applied to the loaded area, with $f(y,z) = 0$ outside the loaded area. Equations (10) and (11) must first be used to produce a self-equilibrating loading $f_1(y,z)$ on the end face.

From (11) the total load is:

$$\begin{aligned} P &= \int_{-b}^{+b}\int_{-c}^{+c} f(y,z)\,dy\,dz \\ &= 16(2k_1b)(2k_2c) + 0 \\ &= 64k_1k_2bc \end{aligned}$$

Applying (10) gives the modified loading function.

$$\begin{aligned} f_1(y,z) &= \left[f(y,z) - \frac{P}{4bc}\right] \\ &= f(y,z) - \frac{64k_1k_2bc}{4bc} \\ &= f(y,z) - 4 \end{aligned}$$

Hence,

$$\begin{aligned} f_1(y,z) &= 16 - 4 = +12 \text{ over the loading area} \\ &= 0 - 4 = -4 \text{ outside the loading area} \end{aligned}$$

This applied stress field is shown in Figure 2.

Sample results from the Direct method are shown in Figure 3. Plots of the bulk stress distribution on the $y = +b$ surface and the end stress σ_x are presented. These results were obtained using seven terms of the Fourier series (i.e. $l, m, n = 0, 1, ...7$), and show good convergence. Shown in Figure 3a is the surface plot of σ_x, with its corresponding values indicated in the contour plot, Figure 3b. These plots show the stress values ranging from -5.0 to $+14.0$ with the majority between -4.0 to $+12.0$. This solution is in good agreement with the applied stress field $f_1(y,z)$. However, a small discrepancy is noticeable around the edges of the surface. The bulk stress distribution σ_b on the $y = +b$ surface is shown in Figures 3c and 3d. As expected, symmetries occur about the $x = 0$ and $z = 0$ axes with zero stress values occuring along the $z = 0$ axis.

4.3 Inverse Results

In the Inverse problem, the bulk stress field, generated by the Direct method, was used to calculate the applied end stress. For comparison, the results of the stress distributions obtained from both the Direct and Inverse methods, at different Fourier indicies, are shown in Figure 6.

Initial program versions produced good Inverse results for all Fourier indices up to the fourth term. However, at higher terms, the Inverse solution was poor. The problem occurred evaluating the double integral of equation (13), using Simpson's rule with up to 161 grid points in each direction. A Gauss-Quadrature method was then implemented, in a general form, so that the number of Gauss points could be increased easily if required. By using 81 grid points in the x and z directions, good results were obtained for up to eight Fourier indicies (i.e. $l, m, n = 0, 1, ...8$). The number of Gauss points used between each grid point is indicated in Figure 4.

It is evident that more Gauss points must be used when solving for a higher number of Fourier coefficients. Figure 5 shows the effect of increasing the number of Fourier coefficients without changing the number of Gauss points used by the integration routine. Figures 5a and 5b show the surface and contour plots produced by the Direct program for $l, m, n = 0, 1, ...9$. Figures 5c and 5d show similar plots produced by the Inverse program. Clearly the agreement between the two solutions is not as good as for the plots presented in Figure 4. Further increases in the number of Gauss points and Fourier coefficients were not warranted for preliminary work due to the large increase in computer resources required.

One disadvantage of the Gauss-Quadrature method is that the bulk stress distribution is required at points which are not evenly spaced over the surface of interest. Since a thermoelastic (i.e. SPATE) measurement is likely to contain stress information for a number of evenly spaced points, the stress values at the location of the Gauss points would first need to be interpolated from the measured values.

5. CONCLUSION

This work has shown that it is possible to predict the stress distribution in a rectangular prism using bulk stress measurements from one surface only. Analysis has been performed for the case where the bulk stress distribution used in the Inverse method was produced by the Direct method. The results obtained were good for up to 8 Fourier indices.

Although the concept has been successfully proven, a considerable amount of work will be required to extend this methodology to a procedure which can be routinely used for the analysis of thermoelastic measurements from real structures.

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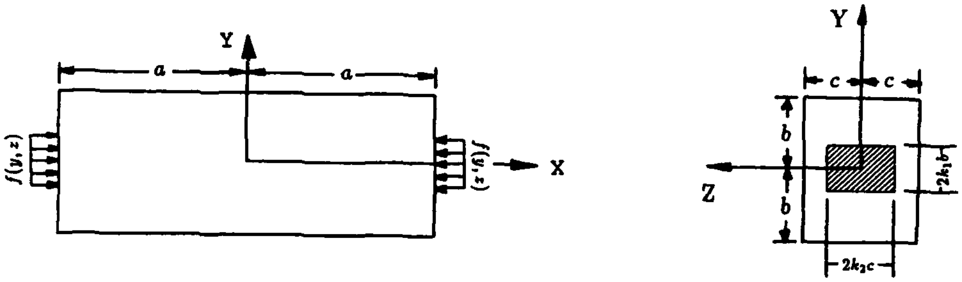


FIGURE 1. RECTANGULAR PRISM WITH CENTRAL LOADING

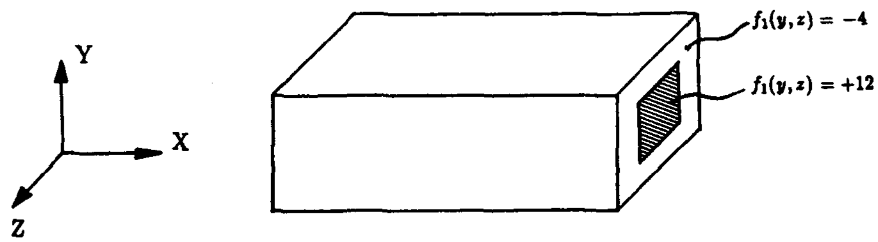


FIGURE 2. ARBITRARILY CHOSEN LOAD CASE

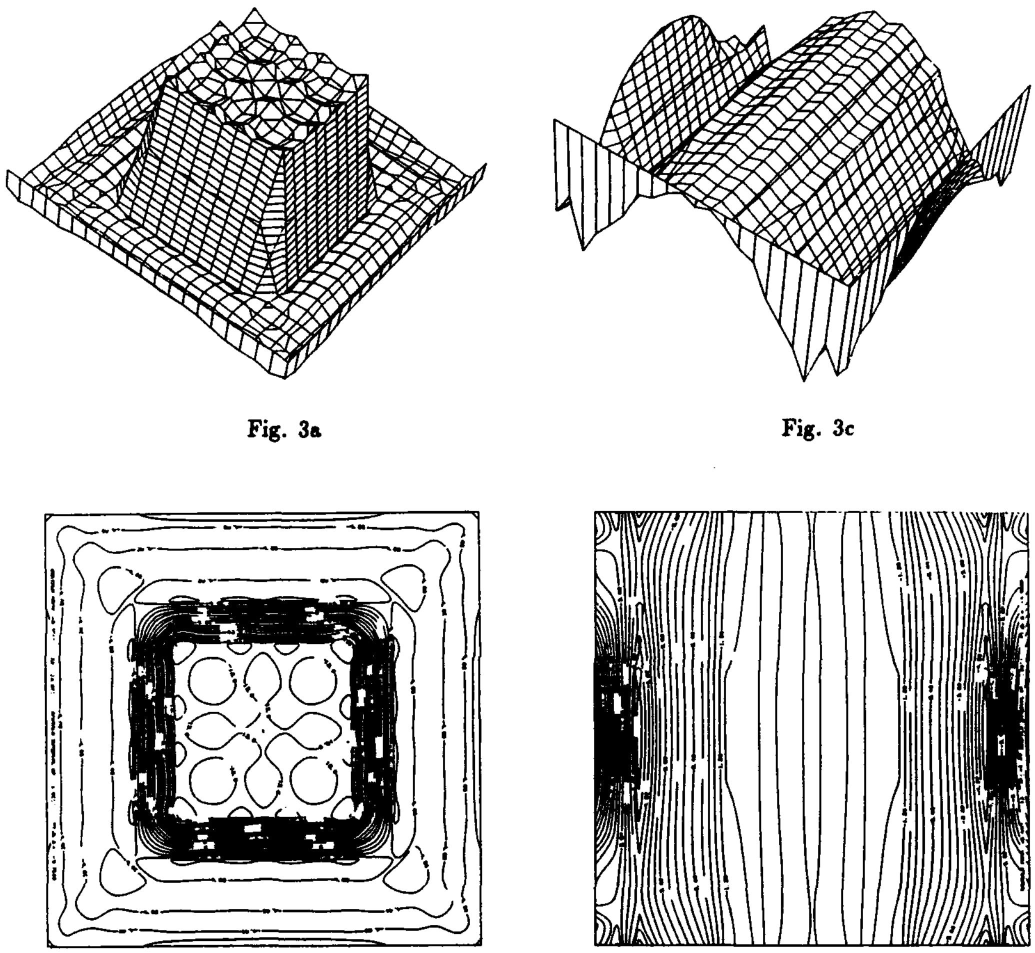


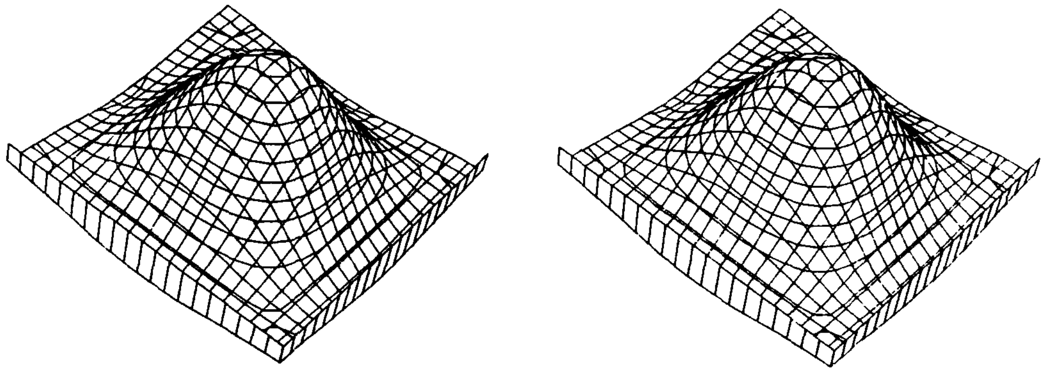
Fig. 3a

Fig. 3c

Fig. 3b

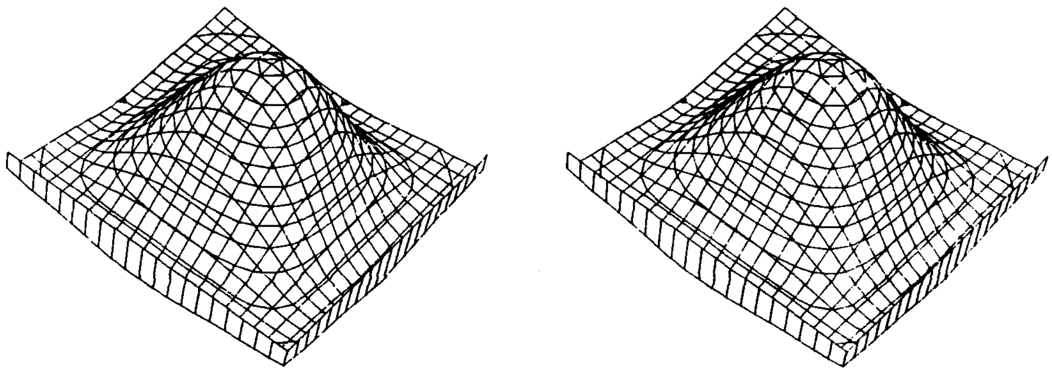
Fig. 3d

FIGURE 3. DIRECT SOLUTION FOR $l, m, n = 0, 1, ...7$



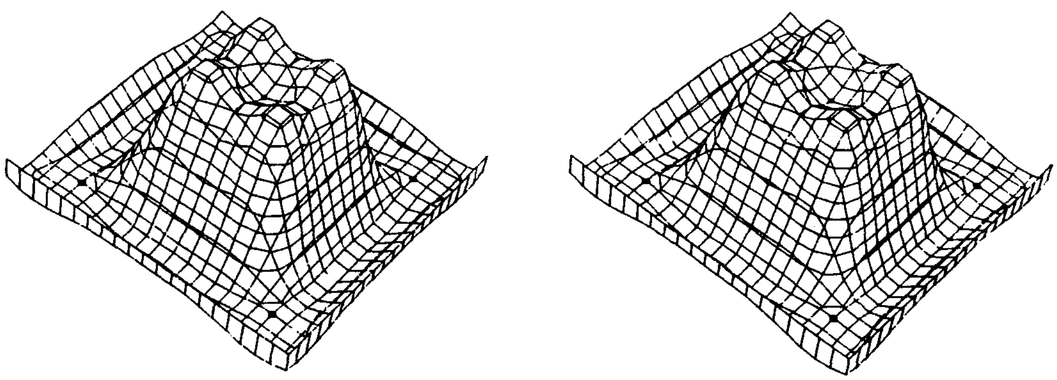
Direct Solution　　　　Inverse Solution

Fig 4a. $l, m, n = 0, 1$. **Two point integration.**



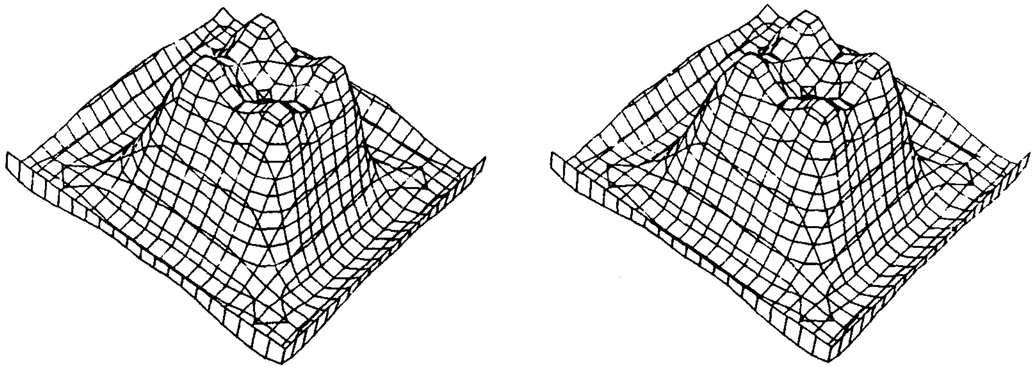
Direct Solution　　　　Inverse Solution

Fig 4b. $l, m, n = 0, 1, 2$. **Two point integration.**



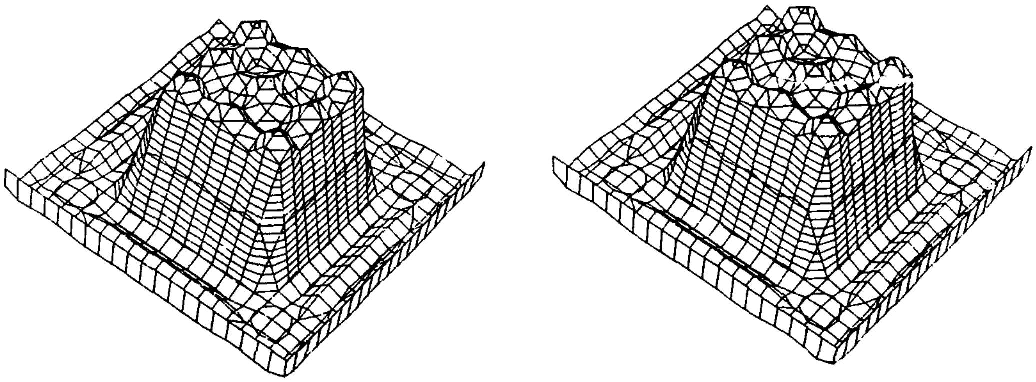
Direct Solution　　　　Inverse Solution

Fig 4c. $l, m, n = 0, 1...3$. **Two point integration.**



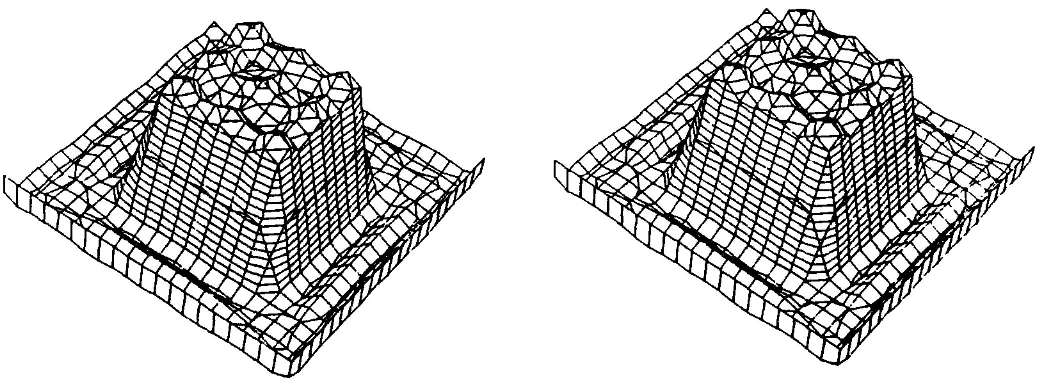
Direct Solution　　Inverse Solution

Fig 4d. $l, m, n = 0, 1 \ldots 4$. Three point integration.



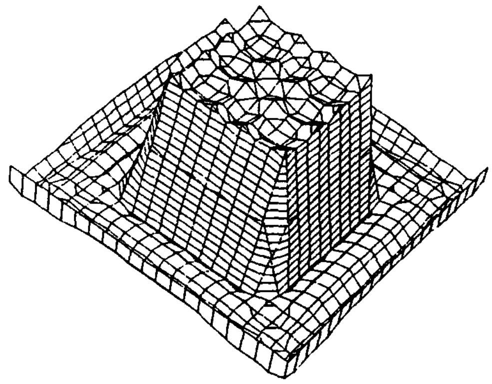
Direct Solution　　Inverse Solution

Fig 4e. $l, m, n = 0, 1 \ldots 5$. Three point integration.

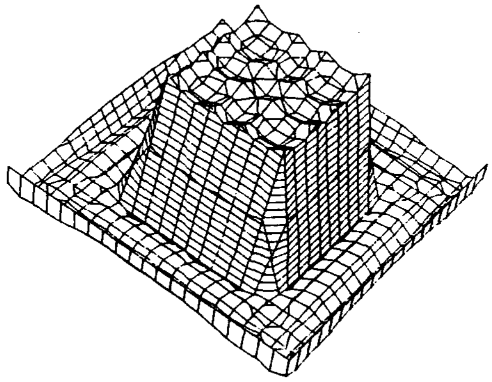


Direct Solution　　Inverse Solution

Fig 4f. $l, m, n = 0, 1 \ldots 6$. Four point integration.

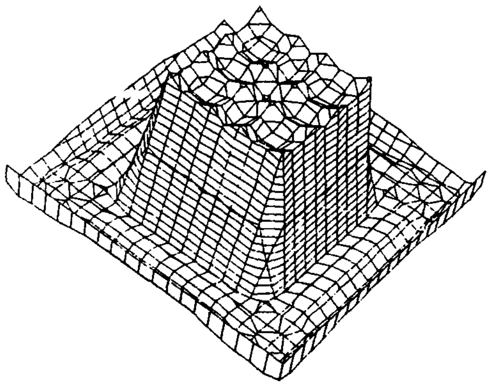


Direct Solution

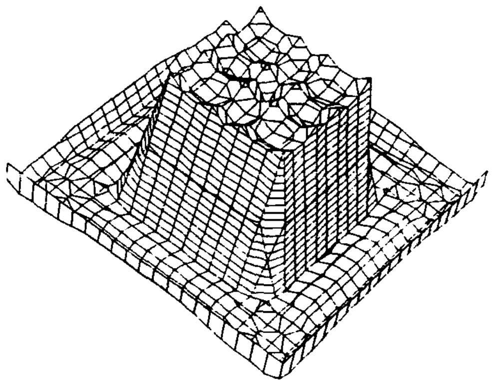


Inverse Solution

Fig 4g. $l, m, n = 0, 1 \ldots 7$. Five point integration.



Direct Solution



Inverse Solution

Fig 4h. $l, m, n = 0, 1 \ldots 8$. Six point integration.

FIGURE 4. DIRECT AND INVERSE SOLUTIONS FOR VARYING l, m, n.

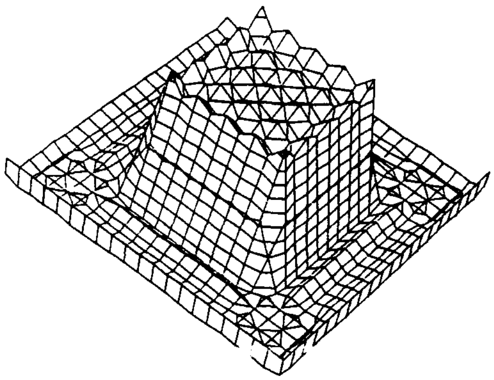


Fig. 5a. Direct Solution

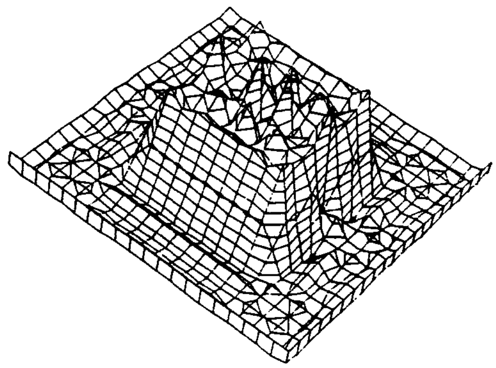


Fig. 5c. Inverse Solution

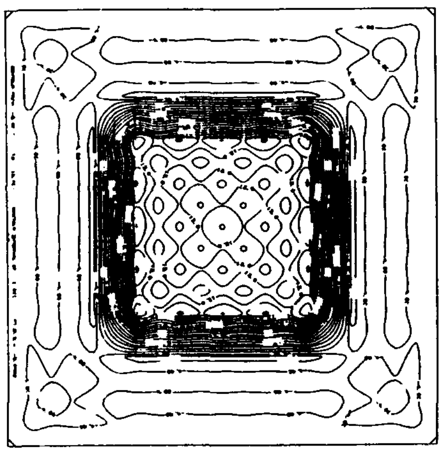


Fig. 5b. Direct Solution

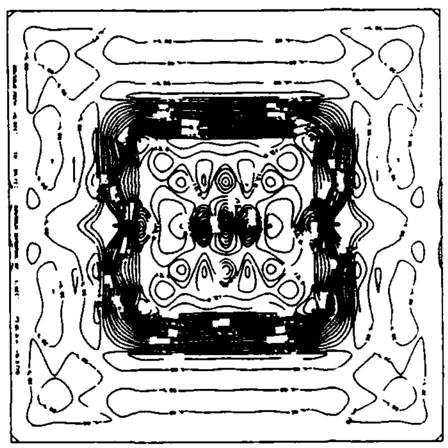


Fig. 5d. Inverse Solution

FIGURE 5. DIRECT AND INVERSE SOLUTION FOR $l, m, n = 0, 1, \ldots 9$

DERIVATION OF EQUATION (13)

From Equation (4)

$$
\begin{aligned}
\sigma_x + \sigma_z = & \sum_m \sum_n \frac{A_{mn}a}{\cosh \alpha_{mn}a} [(1 + \alpha_{mn}a \coth \alpha_{mn}a) \cosh \alpha_{mn}x - \alpha_{mn}x \sinh \alpha_{mn}x] \\
& \times \cos \frac{m\pi y}{b} \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{B_{nl}b}{\beta_{nl}^2 c^2 \cosh \beta_{nl}b} \left[2\nu n^2 \pi^2 \cosh \beta_{nl}y + \left(\frac{l\pi c}{a} \right)^2 \right. \\
& \left. \times [(1 - \beta_{nl}b \coth \beta_{nl}b) \cosh \beta_{nl}y + \beta_{nl}y \sinh \beta_{nl}y] \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \\
& + \sum_l \sum_m \frac{D_{lm}c}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm}c} \left[2\nu m^2 \pi^2 \cosh \gamma_{lm}z + \left(\frac{l\pi b}{a} \right)^2 [(1 - \gamma_{lm}c \coth \gamma_{lm}c) \right. \\
& \left. \times \cosh \gamma_{lm}z + \gamma_{lm}z \sinh \gamma_{lm}z] \right] \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b} \\
& + \sum_l \sum_m \frac{D_{lm}c}{\cosh \gamma_{lm}c} [(1 + \gamma_{lm}c \coth \gamma_{lm}c) \cosh \gamma_{lm}z - \gamma_{lm}z \sinh \gamma_{lm}z] \\
& \times \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b} + \sum_m \sum_n \frac{A_{mn}a}{\alpha_{mn}^2 b^2 \cosh \alpha_{mn}a} \left[2\nu m^2 \pi^2 \cosh \alpha_{mn}x + \left(\frac{n\pi b}{c} \right)^2 \right. \\
& \left. \times [(1 - \alpha_{mn}a \coth \alpha_{mn}a) \cosh \alpha_{mn}x + \alpha_{mn}x \sinh \alpha_{mn}x] \right] \cos \frac{m\pi y}{b} \cos \frac{n\pi z}{c} \\
& + \sum_n \sum_l \frac{B_{nl}b}{\beta_{nl}^2 a^2 \cosh \beta_{nl}b} \left[2\nu l^2 \pi^2 \cosh \beta_{nl}y + \left(\frac{n\pi a}{c} \right)^2 [(1 - \beta_{nl}b \coth \beta_{nl}b) \right. \\
& \left. \times \cosh \beta_{nl}y + \beta_{nl}y \sinh \beta_{nl}y] \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a}
\end{aligned}
$$

On a lateral surface of $y = b$, $\sigma_y = 0$ and so the bulk stress on this surface becomes $\sigma_b = \sigma_x + \sigma_z$. Putting $y = b$ into the above equation yields the following result.

$$\sigma_b = \sum_m \sum_n \frac{A_{mn}a(-1)^m}{\cosh \alpha_{mn} a}[(1 + \alpha_{mn} a \coth \alpha_{mn} a)\cosh \alpha_{mn} x - \alpha_{mn} x \sinh \alpha_{mn} x] \cos \frac{n\pi z}{c} \quad \text{(I)}$$

$$+ \sum_n \sum_l \frac{B_{nl} b}{\beta_{nl}^2 c^2 \cosh \beta_{nl} b}\left[2\nu n^2 \pi^2 \cosh \beta_{nl} b + \left(\frac{l\pi c}{a}\right)^2 [(1 - \beta_{nl} b \coth \beta_{nl} b)\right.$$

$$\left. \times \cosh \beta_{nl} b + \beta_{nl} b \sinh \beta_{nl} b]\right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \quad \text{(II)}$$

$$+ \sum_l \sum_m \frac{D_{lm} c(-1)^m}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm} c}\left[2\nu m^2 \pi^2 \cosh \gamma_{lm} z + \left(\frac{l\pi b}{a}\right)^2 [(1 - \gamma_{lm} c \coth \gamma_{lm} c)\right.$$

$$\left. \times \cosh \gamma_{lm} z + \gamma_{lm} z \sinh \gamma_{lm} z]\right] \cos \frac{l\pi x}{a} \quad \text{(III)}$$

$$+ \sum_l \sum_m \frac{D_{lm} c(-1)^m}{\cosh \gamma_{lm} c}[(1 + \gamma_{lm} c \coth \gamma_{lm} c)\cosh \gamma_{lm} z - \gamma_{lm} z \sinh \gamma_{lm} z] \cos \frac{l\pi x}{a} \quad \text{(IV)}$$

$$+ \sum_m \sum_n \frac{A_{mn} a(-1)^m}{\alpha_{mn}^2 b^2 \cosh \alpha_{mn} a}\left[2\nu m^2 \pi^2 \cosh \alpha_{mn} x + \left(\frac{n\pi b}{c}\right)^2 [(1 - \alpha_{mn} a \coth \alpha_{mn} a)\right.$$

$$\left. \times \cosh \alpha_{mn} x + \alpha_{mn} x \sinh \alpha_{mn} x]\right] \cos \frac{n\pi z}{c} \quad \text{(V)}$$

$$+ \sum_n \sum_l \frac{B_{nl} b}{\beta_{nl}^2 a^2 \cosh \beta_{nl} b}\left[2\nu l^2 \pi^2 \cosh \beta_{nl} b + \left(\frac{n\pi a}{c}\right)^2 [(1 - \beta_{nl} b \coth \beta_{nl} b)\right.$$

$$\left. \times \cosh \beta_{nl} b + \beta_{nl} b \sinh \beta_{nl} b]\right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \quad \text{(VI)}$$

To determine the Fourier coefficients A_{mn}, B_{nl} and D_{lm} it is necessary to take the Fourier transform of the above equation.

$$\int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c}\, dx\, dz$$
$$= \int_{-c}^{+c} \int_{-a}^{+a} \Big[\text{(I)} + \text{(II)} + \text{(III)} + \text{(IV)} + \text{(V)} + \text{(VI)}\Big] \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c}\, dx\, dz$$

Note that p and q are now fixed positive integers.

Integrating the RHS, taking term (I)

$$\int_{-c}^{+c}\int_{-a}^{+a}(\mathrm{I})\cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$

$$=\sum_m\sum_n\frac{A_{mn}a(-1)^m}{\cosh\alpha_{mn}a}\left[(1+\alpha_{mn}a\coth\alpha_{mn}a)\int_{-a}^{+a}\cosh\alpha_{mn}x\cos\frac{p\pi x}{a}\,dx\right.$$
$$\left.-\alpha_{mn}\int_{-a}^{+a}x\sinh\alpha_{mn}x\cos\frac{p\pi x}{a}\,dx\right]\left[\int_{-c}^{+c}\cos\frac{n\pi z}{c}\cos\frac{q\pi z}{c}\,dz\right]$$

$$=\sum_m\frac{A_{mn}ac(-1)^m(1+\delta_{on})}{\cosh\alpha_{mn}a}\left[(1+\alpha_{mn}a\coth\alpha_{mn}a)\frac{2\alpha_{mn}(-1)^p\sinh\alpha_{mn}a}{\alpha_{mn}^2+(p\pi/a)^2}\right.$$
$$\left.-\frac{2\alpha_{mn}^2a(-1)^p\cosh\alpha_{mn}a}{\alpha_{mn}^2+(p\pi/a)^2}+\frac{2\alpha_{mn}(-1)^p\sinh\alpha_{mn}a}{(\alpha_{mn}^2+(p\pi/a)^2)^2}(\alpha_{mn}^2-(p\pi/a)^2)\right]$$

$$=\sum_m\frac{2A_{mn}\alpha_{mn}ac(-1)^{m+p}(1+\delta_{on})}{\alpha_{mn}^2+(p\pi/a)^2}\left[(1+\alpha_{mn}a\coth\alpha_{mn}a)\tanh\alpha_{mn}a\right.$$
$$\left.-\alpha_{mn}a+\tanh\alpha_{mn}a\frac{\alpha_{mn}^2-(p\pi/a)^2}{\alpha_{mn}^2+(p\pi/a)^2}\right]$$

Rearrange and simplify to give :

$$=\sum_m\frac{4A_{mn}\alpha_{mn}^3ac(-1)^{m+p}(1+\delta_{on})}{(\alpha_{mn}^2+(p\pi/a)^2)^2}\tanh\alpha_{mn}a$$

Put $l=p$

$$=\sum_m\frac{4A_{mn}\alpha_{mn}^3ac(-1)^{m+l}(1+\delta_{on})}{(\alpha_{mn}^2+(l\pi/a)^2)^2}\tanh\alpha_{mn}a$$

where

$$\delta_{on}=\begin{cases}1 & \text{when } n=0\\ 0 & \text{when } n\neq 0\end{cases}$$

Taking term (II)

$$\int_{-c}^{+c}\int_{-a}^{+a}(\mathrm{II})\cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$

$$=\sum_n\sum_l\frac{B_{nl}b}{\beta_{nl}^2c^2\cosh\beta_{nl}b}\left[2\nu n^2\pi^2\cosh\beta_{nl}b+\left(\frac{l\pi c}{a}\right)^2[(1-\beta_{nl}b\coth\beta_{nl}b)\right.$$

$$\left.\times\cosh\beta_{nl}b+\beta_{nl}b\sinh\beta_{nl}b]\right]\int_{-a}^{+a}\cos\frac{l\pi x}{a}\cos\frac{p\pi x}{a}\,dx\int_{-c}^{+c}\cos\frac{n\pi z}{c}\cos\frac{q\pi z}{c}\,dz$$

$$=\frac{B_{nl}ab(1+\delta_{ol})(1+\delta_{on})}{\beta_{nl}^2c\cosh\beta_{nl}b}\left[2\nu n^2\pi^2\cosh\beta_{nl}b+\left(\frac{l\pi c}{a}\right)^2[(1-\beta_{nl}b\coth\beta_{nl}b)\cosh\beta_{nl}b+\beta_{nl}b\sinh\beta_{nl}b]\right]$$

$$=\frac{B_{nl}ab(1+\delta_{ol})(1+\delta_{on})}{\beta_{nl}^2c}\left[2\nu n^2\pi^2+\left(\frac{l\pi c}{a}\right)^2[(1-\beta_{nl}b\coth\beta_{nl}b)+\beta_{nl}b\tanh\beta_{nl}b]\right]$$

where

$$\delta_{ol}=\begin{cases}1 & \text{when } l=0\\ 0 & \text{when } l\neq 0\end{cases}$$

$$\delta_{on}=\begin{cases}1 & \text{when } n=0\\ 0 & \text{when } n\neq 0\end{cases}$$

Taking term (III)

$$\int_{-c}^{+c}\int_{-a}^{+a}(\mathrm{III})\cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$

$$=\sum_l\sum_m\frac{D_{lm}c(-1)^m}{\gamma_{lm}^2b^2\cosh\gamma_{lm}c}\left[\left\{2\nu m^2\pi^2+\left(\frac{l\pi b}{a}\right)^2(1-\gamma_{lm}c\coth\gamma_{lm}c)\right\}\right.$$

$$\left.\times\int_{-c}^{+c}\cosh\gamma_{lm}z\cos\frac{q\pi z}{c}\,dz+\left(\frac{l\pi b}{a}\right)^2\gamma_{lm}\int_{-c}^{+c}z\sinh\gamma_{lm}z\cos\frac{q\pi z}{c}\,dz\right]$$

$$\times\int_{-a}^{+a}\cos\frac{l\pi x}{a}\cos\frac{p\pi x}{a}\,dx$$

$$=\sum_m\frac{D_{lm}ca(-1)^m(1+\delta_{ol})}{\gamma_{lm}^2b^2\cosh\gamma_{lm}c}\left[\left\{2\nu m^2\pi^2+\left(\frac{l\pi b}{a}\right)^2(1-\gamma_{lm}c\coth\gamma_{lm}c)\right\}\right.$$

$$\times\frac{2\gamma_{lm}(-1)^q\sinh\gamma_{lm}c}{\gamma_{lm}^2+(q\pi/c)^2}+\left(\frac{l\pi b}{a}\right)^2\gamma_{lm}\frac{2\gamma_{lm}c(-1)^q\cosh\gamma_{lm}c}{\gamma_{lm}^2+(q\pi/c)^2}$$

$$\left.-\left(\frac{l\pi b}{a}\right)^2\gamma_{lm}2(-1)^q\sinh\gamma_{lm}c\frac{\gamma_{lm}^2-(q\pi/c)^2}{\gamma_{lm}^2+(q\pi/c)^2}\right]$$

$$=\sum_m\frac{D_{lm}2ac(-1)^{m+q}(1+\delta_{ol})}{\gamma_{lm}b^2(\gamma_{lm}^2+(q\pi/c)^2)}\left[2\nu m^2\pi^2\tanh\gamma_{lm}c+\left(\frac{l\pi b}{a}\right)^2(1-\gamma_{lm}c\coth\gamma_{lm}c)\right.$$

$$\left.\times\tanh\gamma_{lm}c+\left(\frac{l\pi b}{a}\right)^2\gamma_{lm}c-\left(\frac{l\pi b}{a}\right)^2\frac{\gamma_{lm}^2-(q\pi/c)^2}{\gamma_{lm}^2+(q\pi/c)^2}\tanh\gamma_{lm}c\right]$$

Put $n = q$, re-arrange and simplify to give:

$$= \sum_m \frac{4D_{lm}ac(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{\gamma_{lm}b^2\left(\gamma_{lm}^2 + (n\pi/c)^2\right)}\left[\nu m^2\pi^2 + \frac{(l\pi b/a)^2(n\pi/c)^2}{\gamma_{lm}^2 + (n\pi/c)^2}\right]$$

where

$$\delta_{ol} = \begin{cases} 1 & \text{when } l = 0 \\ 0 & \text{when } l \neq 0 \end{cases}$$

The integrals of parts (IV) (V) and (VI) were derived using the same approach as shown in (I) (II) and (III).

$$\int_{-c}^{+c}\int_{-a}^{+a} (\mathrm{IV}) \cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$
$$= \sum_m \frac{4D_{lm}\gamma_{lm}^3 ac(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{\left(\gamma_{lm}^2 + (n\pi/c)^2\right)^2}$$

$$\int_{-c}^{+c}\int_{-a}^{+a} (\mathrm{V}) \cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$
$$= \sum_m \frac{4A_{mn}ac(-1)^{m+l}(1+\delta_{on})\tanh\alpha_{mn}a}{\alpha_{mn}b^2\left(\alpha_{mn}^2 + (l\pi/a)^2\right)}\left[\nu m^2\pi^2 + \frac{(n\pi b/c)^2(l\pi/a)^2}{\alpha_{mn}^2 + (l\pi/a)^2}\right]$$

$$\int_{c}^{+c}\int_{-a}^{+a} (\mathrm{VI}) \cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$
$$= \frac{B_{nl}bc(1+\delta_{ol})(1+\delta_{on})}{\beta_{nl}^2 a}\left[2\nu l^2\pi^2 + \left(\frac{n\pi a}{c}\right)^2\left[(1 - \beta_{nl}b\coth\beta_{nl}b) + \beta_{nl}b\tanh\beta_{nl}b\right]\right]$$

where

$$\delta_{ol} = \begin{cases} 1 & \text{when } l = 0 \\ 0 & \text{when } l \neq 0 \end{cases}$$

$$\delta_{on} = \begin{cases} 1 & \text{when } n = 0 \\ 0 & \text{when } n \neq 0 \end{cases}$$

The individual integral components may now be summed to give the following.

$$\int_{-c}^{+c}\int_{-a}^{+a}\Big[(\mathrm{I})+(\mathrm{II})+(\mathrm{III})+(\mathrm{IV})+(\mathrm{V})+(\mathrm{VI})\Big]\cos\frac{p\pi x}{a}\cos\frac{q\pi z}{c}\,dx\,dz$$
$$=\sum_m \frac{4A_{mn}\alpha_{mn}^3 ac(-1)^{m+l}(1+\delta_{on})\tanh\alpha_{mn}a}{(\alpha_{mn}^2+(l\pi/a)^2)^2}$$
$$+\sum_m \frac{4A_{mn}ac(-1)^{m+l}(1+\delta_{on})\tanh\alpha_{mn}a}{\alpha_{mn}b^2(\alpha_{mn}^2+(l\pi/a)^2)}\left[\nu m^2\pi^2+\frac{(n\pi b/c)^2(l\pi/a)^2}{\alpha_{mn}^2+(l\pi/a)^2}\right]$$
$$+\frac{B_{nl}ab(1+\delta_{ol})(1+\delta_{on})}{\beta_{nl}^2 c}\left[2\nu n^2\pi^2+\left(\frac{l\pi c}{a}\right)^2[(1-\beta_{nl}b\coth\beta_{nl}b)+\beta_{nl}b\tanh\beta_{nl}b]\right]$$
$$+\frac{B_{nl}bc(1+\delta_{ol})(1+\delta_{on})}{\beta_{nl}^2 a}\left[2\nu l^2\pi^2+\left(\frac{n\pi a}{c}\right)^2[(1-\beta_{nl}b\coth\beta_{nl}b)+\beta_{nl}b\tanh\beta_{nl}b]\right]$$
$$+\sum_m \frac{4D_{lm}ac(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{\gamma_{lm}b^2(\gamma_{lm}^2+(n\pi/c)^2)}\left[\nu m^2\pi^2+\frac{(l\pi b/a)^2(n\pi/c)^2}{\gamma_{lm}^2+(n\pi/c)^2}\right]$$
$$+\sum_m \frac{4D_{lm}\gamma_{lm}^3 ac(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{(\gamma_{lm}^2+(n\pi/c)^2)^2}$$

Simplifying this equation gives the following result, as presented in equation (13).

$$\frac{1}{abc}\int_{-c}^{+c}\int_{-a}^{+a}(\sigma_x+\sigma_z)\big|_{y=b}\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{n\pi z}{c}\right)dx\,dz$$
$$=\sum_m \frac{4A_{mn}(-1)^{m+l}(1+\delta_{on})\tanh\alpha_{mn}a}{b\left(\alpha_{mn}^2+(l\pi/a)^2\right)^2}$$
$$\times\left[\alpha_{mn}^3+\frac{1}{\alpha_{mn}b^2}\left\{\nu m^2\pi^2\left(\alpha_{mn}^2+\left(\frac{l\pi}{a}\right)^2\right)+\left(\frac{n\pi b}{c}\right)^2\left(\frac{l\pi}{a}\right)^2\right\}\right]$$
$$+\sum_m \frac{4D_{lm}(-1)^{m+n}(1+\delta_{ol})\tanh\gamma_{lm}c}{b\left(\gamma_{lm}^2+(n\pi/c)^2\right)^2}$$
$$\times\left[\gamma_{lm}^3+\frac{1}{\gamma_{lm}b^2}\left\{\nu m^2\pi^2\left(\gamma_{lm}^2+\left(\frac{n\pi}{c}\right)^2\right)+\left(\frac{l\pi b}{a}\right)^2\left(\frac{n\pi}{c}\right)^2\right\}\right]$$
$$+\frac{B_{nl}\pi^2}{\beta_{nl}^2a^2c^2}(1+\delta_{on})(1+\delta_{ol})(a^2n^2+c^2l^2)$$
$$\times(2\nu+1-\beta_{nl}b\coth\beta_{nl}b+\beta_{nl}b\tanh\beta_{nl}b)$$

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16. ABSTRACT

A mathematical method is presented for the determination of the normal symmetric end loads from known values of the bulk stress on a lateral surface of a three dimensional rectangular prism. A FORTRAN computer program has been developed to implement this method, and results for an arbitrarily chosen case are presented.

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